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Abstract
In a recent paper by McClelland (2011 *Phys. Educ.* **46** 469–71) some situations of locomotion are examined, and the assertion 'The person is accelerated by a net force from the Earth' is considered invalid. Arguments are presented to demonstrate that this assertion is correct and to analyse the corresponding difficulties.

Introduction

This note is a response to a recent paper by McClelland [1]. The situation of a person who is walking horizontally along the ground is examined, and the statement 'The person is accelerated by a net force from the Earth' is considered invalid. A key point in McClelland's analysis is his claim that: 'The force responsible for acceleration of any real body acts at an identifiable point or region on the body and that region accelerates in the direction of the force. It is absurd to suggest that it would accelerate in the opposite direction'. This argument is, indeed, the basis of McClelland's critique of what he calls 'a very persistent mistake'.

This is not a trivial matter, because denying the accelerating role of the force exerted by the Earth on the pedestrian is equivalent to denying the consistency of Newton's laws. In what follows I shall attempt to make the argument as clear as possible, and to discuss why the consequences of Newtonian mechanics are not, as here, always in agreement with intuition.

Newton's laws for one and several particles

Let there be some particles (i), each one of invariable mass m_i, each undergoing a total force $\boldsymbol{f}_i$, with pairs of particles exerting mutual forces $\boldsymbol{f}_{ij}$ (i on j) and $\boldsymbol{f}_{ji}$ (j on i).

In a Galilean frame of reference, we can write Newton's laws for each particle, with quantities all taken at the same moment of time:

(second law) $\boldsymbol{f}_i = m_i\boldsymbol{a}_i$ or $\boldsymbol{f}_i = \mathrm{d}\boldsymbol{p}_i/\mathrm{d}t$ with $\boldsymbol{a}_i$, $\boldsymbol{p}_i$ respectively being the acceleration and momentum of particle i

(third law) $\boldsymbol{f}_{ij} = -\boldsymbol{f}_{ji}$.

These relationships are valid at all times as long as the speeds are negligible in comparison to the speed of light.

If some particles are grouped in a system of mass M, with $M = \sum_i m_i$, the net force acting on a particle is $\sum \boldsymbol{f}_i = \sum \boldsymbol{f}_{\mathrm{ext}-i} + \sum \boldsymbol{f}_{\mathrm{int}-i}$ where $\sum \boldsymbol{f}_{\mathrm{ext}-i}$ is the total force exerted on particle i by all the particles which are outside the system and $\sum \boldsymbol{f}_{\mathrm{int}-i}$ is the total force exerted on particle i by all the other particles of the system: $\sum \boldsymbol{f}_{\mathrm{int}-i} = \sum_j \boldsymbol{f}_{ji}$.

Writing the second law for each particle i and adding term by term the corresponding relationships leads to $\sum_i \boldsymbol{f}_i = \sum_i m_i\boldsymbol{a}_i$.

Given the definition of the centre of mass of the system, G, the quantity $\sum_i m_i\boldsymbol{a}_i$ equals $M\boldsymbol{a}_G$, where $\boldsymbol{a}_G$ is the acceleration of G.

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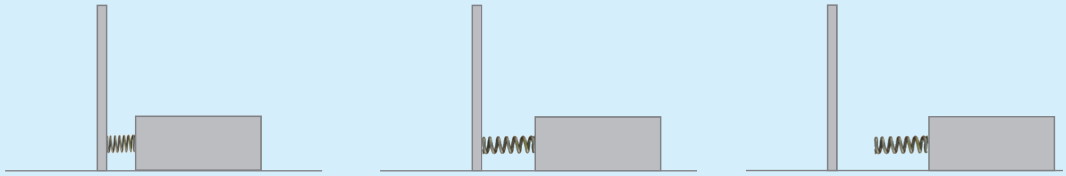

(a) A rigid body attached to a compressed spring
—can slide without friction on a horizontal support,
—is posed against a wall rigidly connected to the support and the Earth.

(b) Once the string is cut
—the spring extends;
—as long as the spring is in contact with the wall, the body and the spring are accelerated away from the wall.

(c) Once the spring is no longer in contact with the wall, the body and the spring keep their motion away from the wall, at constant speed.

Figure 1. A model situation.

What about $\sum_i \boldsymbol{f}_i$? This quantity turns out to be only related to external forces.

Indeed, $\sum_i \boldsymbol{f}_i = \sum_i \boldsymbol{f}_{\text{ext}-i} + \sum_i \boldsymbol{f}_{\text{int}-i}$.

But $\sum_i \boldsymbol{f}_{\text{int}-i} = 0$. This is due to the third law: $\boldsymbol{f}_{ij} = -\boldsymbol{f}_{ij}$, so that all the pairs of internal forces balance out.

Finally, $\sum_i \boldsymbol{f}_i = \sum_i \boldsymbol{f}_{\text{ext}-i}$ and (with $\boldsymbol{F}_{\text{ext}} = \sum_i \boldsymbol{f}_{\text{ext}-i}$) we can write $\boldsymbol{F}_{\text{ext}} = M\boldsymbol{a}_G$ or equivalently $\boldsymbol{F}_{\text{ext}} = \frac{\text{d}\boldsymbol{p}}{\text{d}t}$, with $\boldsymbol{p}$ being the total momentum of the system[1]. For the same reason, we can also write the third law globally for two systems: the total force exerted by a system, A, on another system, B, is opposed to the total force exerted by system B on system A.

All this is valid whether the system(s) is (are) deformable or not, provided, of course, that we do not change the particles defining a given system.

By contrast, the deformability of a system makes a difference where the kinetic energy is concerned. For each particle i, the work of the force $\boldsymbol{f}_i$ with a displacement $\boldsymbol{dl}_i$ of its point of application equals the change in kinetic energy of this particle during the same time interval. Adding term by term the corresponding relationships for all the particles of a system gives: $\sum_i \boldsymbol{f}_i \cdot \boldsymbol{dl}_i = \Delta E_{\text{k}}$, where ΔE_{k} is the change in total kinetic energy of the system.

This time, all the forces are relevant, because the work of internal forces may be non-zero. This is the case for a deformable body. Therefore it is essential to remember that, concerning the kinetic energy, we have to take into account the work done by all the forces, the internal as well as the external: $\Delta E_{\text{k}} = \sum_i \boldsymbol{f}_i \cdot \boldsymbol{dl}_i = \sum_i \boldsymbol{f}_{\text{ext}-i} \cdot \boldsymbol{dl}_i + \sum_i \boldsymbol{f}_{\text{int}-i} \cdot \boldsymbol{dl}_i$.

With this in mind, plus of course the need not to change the particles of a given system, we are ready to cope with situations of deformation without any problem of inconsistency.

Interaction between deformable bodies: a model situation

Imagine a spring that is maintained compressed by a string, with one end just posed against a vertical wall and the other end attached to a rigid object (A) of mass M (figure 1a): M is large with respect to the mass of the spring. The spring and rigid object are situated on a horizontal support on which they can slide without friction. They constitute system A, of mass M_{A}. System B, of mass M_{B}, is constituted of the wall, the building and the Earth. M_{A} is obviously very small with respect to M_{B}.

When we cut the string (figure 1(b)) we see the following.

- The spring and wall stay in contact for a time interval Δt. During this time interval, they exert opposite forces on each other. Then, they separate.
- In the Galilean frame of reference, where systems A and B were initially at rest, the total momentum of system A combined with system B is unchanged, therefore the momenta of the two systems (A and B) are either zero or equal and opposite.
- In this Galilean frame of reference, during the time Δt, the centre of mass G_{A} of system A, and therefore the rigid body, is accelerated away from the wall. System B's

[1] In France, this relationship has a special name: 'Théorème du centre d'inertie'.

acceleration is negligible with respect to that of system A. However, these two accelerations are in opposite directions, and the point of application of the force exerted by the wall on the spring is, strictly speaking, displaced in a direction opposite to that of G_A.

- System A, and in particular the rigid body, acquires kinetic energy. The kinetic energy acquired by system B is negligible in comparison to that of system A.
- Clearly the work that is effective to increase the kinetic energy of system A is that of the internal forces of this system.

There is no particular inconsistency in all these statements, given Newton's laws.

The horizontal components of forces in play when a person accelerates to walk on the ground conform to this analysis, the 'wall' being constituted, this time, by the irregular materials on the ground, which act as small starting blocks. During a phase of acceleration, the person would use his/her leg as a deformable 'spring'.

Starting from this model situation, we can change the mass of system B, by taking another body of mass of the same order as mass M_A , that can also slide without friction on the horizontal support. Only what concerns negligible quantities will be changed with respect to the model situation. But the essential features—opposite forces between the two systems, backward motion of the application point for force 'B on A', and the crucial role of the work done by internal forces—stay the same. Two skaters pushing against one another can be dealt with in this framework, possibly this time with a significant amount of work being done by internal forces in *both* interacting systems (skaters).

Corresponding difficulties

It is one thing to say that there are no inconsistencies with a Newtonian analysis of a given situation, but it is another matter entirely to claim that there are no difficulties in it. Indeed here there are plainly some very substantial difficulties in grasping what is going on.

Let us come back to McClelland's statement, cited in the introduction: 'The force responsible for acceleration of any real body acts at an identifiable point or region on the body and that region accelerates in the direction of the force. It is absurd to suggest that it would accelerate in the opposite direction'. In fact, the simple model situation is incompatible with McClelland's statement. Indeed, the force acting on system A (spring + rigid body) is in the same direction as the acceleration $\boldsymbol{a}_G$ of the centre of mass of A, G_A, as required by Newton's second law. But, and this is crucial, these laws do not require that any 'region' of the concerned system, not even the 'region' on which the external force is acting, must move in the same direction. McClelland writes 'In fact, N (the external force on the propelled body) cannot possibly be the net force accelerating the body as its point of application accelerates in the wrong direction'. But for different subparts of a system, there is no 'wrong direction' that would be deducible from a total external force.

What makes this point so difficult to accept?

Language problems . . . only?

Perhaps we can try to explain how problematic it is to come to an agreement about this situation. The reminder of Newton's laws by McClelland is widely similar to the statements recalled in this paper. It is even acknowledged that 'a 'body' cannot accelerate itself but must interact with something outside itself'. What kinds of obstacle stand in the way of an easy understanding of this phenomenon?

A part of the problem might be the meaning ascribed to several statements.

- 'The 'body' in *a* is not the same as the 'body' in *c*'. With this statement, where *a* and *c* designate two other statements, we might understand that the deformation of the walking person invalidates its permanency as a system. What is it to be 'the same' for a system? In a Newtonian analysis, the permanency of a system only means that the system is constituted of the same particles. This is the case for systems A and B in our model situation.
- What is it to 'accelerate' something, or to have 'an active role in locomotion'? If someone considers that saying 'the Earth exerts a force that accelerates the person'

means that no other forces are crucial to produce a motion, then he/she may well have a problem, because, in terms of energy, we need there to be some forces associated with a non-zero amount of work. If we read this statement as follows: 'in this situation of acceleration, there is necessarily a (forward) force from the Earth acting on the person, and it is related to the acceleration by the second law', there is no inconsistency. Moreover, we are then thinking correctly in the spirit of Newton's laws, as strictly speaking Newton's laws are not causal, since they involve quantities 'taken at the same moment of time'.

Similarly, there may be a problem with the meaning of the expression 'to push'. This verb might be understood as implying a displacement of the point of application of the propelling force in its own direction. Think of the arrows in comics, which are evocative both of force and of motion. Then, we could not say that the Earth pushes the pedestrian. But if 'to push' means only to exert a force on a body in the same direction as that of the acceleration of its centre of mass, then we can accept this last statement.

The moral of this story is that scientific language is useful, because it is much less ambiguous than our everyday language. There may be another reason, which I suspect to be the main one, for the difficulties mentioned above. It is the strong tendency we all have to reason locally; more precisely to reason with a local causality ([2], chapter 5). We tend to expect that a force exerted on a 'region', as McClelland puts it, will produce an identifiable effect, that is to say a motion in the same direction. As shown above, concerning deformable systems, Newton's laws do not confirm this expectation, even if this seems 'absurd' to our intuition. Note in particular that the points of application of the external forces exerted on a given system are totally irrelevant when we just consider what happens to the centre of mass of this system ([2], chapter 10). One more counterintuitive statement.

Final comment

It may seem regrettable that Newton's laws are so counterintuitive, despite the fact that they may seem, at first sight, to be compatible with a simple localized causality. Everyday experience should not be lost sight of, but it is at the same time dramatically opaque and ineffective if we expect it to lead us to a correct Newtonian analysis. In order to avoid inconsistencies, there is no other route than a strict respect of definitions and of the meaning of Newtonian relationships. This is well known, but at the same time we have endlessly to rediscover its importance. We are sometimes surprised—when faced with unexpected effects such as the irrelevance of application points of forces exerted on a system concerning the motion of the centre of mass of a system—that the elegance and parsimony of Newton's laws often force us to accept counterintuitive conclusions.

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