'Spontaneous' ways of reasoning in elementary kinematics

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programmable pocket calculators. The essential point to observe is that the computational formalism need not be understood line for line. The main purpose in introducing this topic is to generate the excitement and fascination of interpreting experimental observations with the use of a theoretical model, and to show how one brings theory into closer accord with reality by progressive improvement of the model. It can thus be convincingly demonstrated that physics is something more than just an incoherent intellectual game with models.

Although the objective of scientific theories is well demonstrated here, this is not the only problem in physics which can generate enthusiasm and provide stimulation. The methods of didactics occupy a special place in gaining access to the fundamental questions of modern solid state physics. For instance, a didactic approach could be taken to the relationship between optical absorption and electrical conductivity. The theoretical concepts of the band model can be related to the experimental data in a way similar to the potential model of F-centres (Kuhn 1979a). For example, it is possible in this way to determine the energy gap between the valence and conduction bands by measuring the light absorption of a CdS crystal. In practice luminescence phenomena and lattice defects in semiconductors find many useful applications in modern technology.

As a further didactic application, the one-dimensional potential well model can be easily extended to three dimensions to provide important access to the orbital model of the hydrogen atom and its application to chemistry (Kuhn 1979b).

Finally, on the basis of the general methodological implication which emerges from the potential well model, the proposed treatment of the colour centres should be recognised as a valuable contribution to that most important of didactic tasks—simplification of presentation. With regard to 'perfection', however, we should like to remain with the sentiment of the introductory remark.

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'Spontaneous' ways of reasoning in elementary kinematics

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Abstract The aim of this study was to explore and analyse 'spontaneous' ways of reasoning (SWR) of students in elementary kinematics (uniform motion in galilean frames). A set of experiments presented to 80 eleven-year-old children and to some 700 first- and fourth-year university students showed types of right and wrong answer which varied little from one sample of pupils to another. It seems difficult to attribute these results solely to school learning; but they can be well accounted for if we assume the existence of an organised system which we call the 'natural model', as opposed to the kinematic model of the physicists. This model involves two components which always interact: a purely descriptive one describing motion, and a causal one explaining motion.

The causal component stems from the permanence of a link between motion and its causes. Motion and velocity are considered as permanent physical properties of the moving body alone, independent of observers, and tend to be defined through reference to the driving forces which cause them, and not to frames. Although situations with drag exemplify the need to distinguish between different velocities for different frames, students never think in terms of frames, and combine 'causes' of motion with one another as forces and not as kinematic velocities: the concepts of force and velocity are not very clearly distinguished.

The descriptive component involves two quite different aspects. The first affects travelled distances and trajectories, which are 'frozen' in a unique and purely geometrical space, independent of observers. This 'geometrisation' of motion results from the elimination of temporal considerations. The second concerns the viewpoints of various observers. Students consider two kinds of motion: 'true' or 'real' motion which is intrinsic because it has a recognised dynamical cause, and 'apparent' motion which is perceived as an optical illusion.

Some consequences for teaching can be drawn from these results, and some pedagogical suggestions are made to help students to overcome their difficulties.

Résumé Cette étude a pour but d'explorer et d'analyser les raisonnements 'spontanés' des étudiants en cinématique élémentaire (mouvements uniformes et référentiels galiléens). Une série d'expériences portant sur 80 enfants de 11 ans et un peu plus de 700 étudiants de première et quatrième année d'université montre que l'on trouve chez les uns et les autres des types de réponses (justes ou fausses) très similaires. Ces résultats peuvent difficilement être attribuées à l'enseignement scolaire seul: en revanche, il est possible d'en rendre compte en supposant l'existence d'un système organisé que nous avons appelé 'modèle naturel' par opposition au modèle cinématique du physicien. Ce modèle présente deux aspects différents qui, cependant, interfèrent en permanence: le premier est purement descriptif et le second causal ou explicatif.

L'aspect causal trouve son origine dans l'existence d'une association stable entre cause motrice et mouvement. Le mouvement et la vitesse sont conçus comme étant une propriété physique de l'objet mobile seul, indépendante des observateurs: ils sont définis à partir des causes motrices qui leur donnent naissance et non dans des référentiels. Bien que les situations avec entraînement favorisent la reconnaissance de vitesses différentes dans des référentiels différents, les étudiants ne raisonnent jamais en termes de référentiels et combinent 'les causes' de mouvements les unes avec les autres comme des forces et non comme des vitesses: les concepts de forces et de vitesse sont très peu différenciés.

L'aspect descriptif comprend deux volets distincts. Le premier concerne les distances parcourues et les trajectoires qui sont 'figées' dans un espace unique, purement géométrique et indépendant des observateurs. Cette géométrisation des mouvements résulte de l'élimination du temps. Le second concerne les points de vue de différents observateurs. Les étudiants considèrent deux sortes de mouvements: les mouvements 'vrais' ou 'réels' qui sont intrinsèques car ils ont une cause dynamique identifiable, et les mouvements 'apparents' qui sont perçus comme des fictions ou des illusions d'optique.

Quelques conséquences pédagogiques peuvent être tirées de ces résultats qui permettent de faire quelques suggestions susceptibles d'aider les étudiants à surmonter leurs difficultés.

1 Introduction

It is well known that, from year to year, students frequently repeat the same errors when dealing with basic problems. The study we report here was undertaken to test the hypothesis that these common difficulties actually reveal a 'natural' or 'spontaneous'† way of reasoning, which is not at all erratic but, on the contrary, rather well structured. This spontaneous way of reasoning (SWR) develops before teaching is offered, and is essentially independent of the various items of knowledge learnt at school, with which it may co-exist to a large extent and for a considerable time. It should not be considered as a distorted 'reinterpretation' of this school knowledge, however, but as an original mental construction.

We attempt to bring SWR to light in the restricted domain of elementary kinematics. It seems clear that, if our hypothesis concerning its existence and influence is correct, an understanding of its characteristics is necessary if teaching methods are to be devised to provide an efficient counteraction.

We must stress that, to disclose SWR, it is necessary to relinquish any attempt to obtain 'correct' answers from students: what is sought is the interpretation, built up by students themselves, of a given problem, the concepts and representations upon which their logic is founded, and the line of argument they follow, whatever the solution they finally obtain.

2 Field of investigation

We studied the attitudes of students towards problems involving uniform motion in changing galilean frames of reference. For the physicist, describing motion implies that the frame to which the motion refers must be specified; the motion is studied by observers attached to this frame who use a stock of adequate rules and clocks which enable them to perform measurements and quantitative descriptions. This is already different from the everyday situation in which 'references' are instead landmarks depending on the relative positions of observers and which is essentially a purely spatial notion, without the intervention of time considerations; thus reference frames (in the physicist's sense) are seldom or never explicitly defined, and qualitative descriptions are ultimately needed.

The mathematical notions and relations required to solve the class of simple problems we studied (uniform motion in galilean frames) are very simple and few in number:

(a) The galilean invariance of lengths and 'instantaneous distances' (that is, distances between the positions of moving bodies at the same instant)

$$l_A = l_B. \qquad (1)$$

(b) The law of velocity composition

$$\boldsymbol{v}_B(M) = \boldsymbol{v}_A(M) + \boldsymbol{v}_B(A) \qquad (2)$$

where $\boldsymbol{v}_A(M)$, for instance, stands for the velocity of body M in the reference frame of observer A.

(c) The galilean invariance of time

$$\Delta t_A = \Delta t_B, \qquad t_A = t_B. \qquad (3)$$

(d) The law of composition of displacements, a useful consequence of (2) and (3), seldom stated in textbooks,

$$\boldsymbol{d}_B(M) = \boldsymbol{d}_A(M) + \boldsymbol{d}_B(A), \qquad (4)$$

where $\boldsymbol{d}_A(M)$, for instance, represents the distance travelled by body M in the frame of observer A.

The mathematical skill necessary to transform these relations to solve the problems considered below is very limited, and the lack of such skill

† 'Natural' or 'spontaneous' here means 'independent of teaching', either at school or university level.

cannot by itself explain the failures observed. On the other hand, some of the concepts involved are far from simple. The notion of the reference frame, with the transportable clocks and rules used by the observers, implies several physical properties which extend far beyond the boundaries of kinematics. And the very idea that motion can only be defined relative to a frame, all galilean frames being equivalent, contradicts the common recognition of rest as opposed to motion, and the commonly used expressions 'true motion' as opposed to 'apparent motion'. Are these related only to a linguistic habit? Do they not reveal a conceptual distinction which is in direct contradiction of the galilean model which is to be learnt?

3 Methodology

A preliminary investigation was organised among 50 first-year university students. They were given three exercises, which were actually the same problem in different guises: motion is observed by two observers, themselves in relative motion—are the velocities and the distances travelled identical for both observers?

A negative answer is systematically expected. Nevertheless:

(i) A high percentage of errors appears (up to 36% for velocities and 63% for distances for a given exercise; for all exercises 60% of students made at least one error for velocities and 80% at least one error for distances).

(ii) Answers depend strongly on the physical context.

(iii) Moreover, when a student gives different answers to the same question, depending on the context, and is urged to recognise the similarities between the exercises, he will often fail to do so, or if he does realise these similarities he will nevertheless stick to his original idea regarding velocities and distances.

(iv) It appears that anwers for velocities and answers for distances are largely uncorrelated.

Clearly, students frequently fail to master the galilean kinematic model with which teaching has provided them—this is really no surprise—but their errors are context-dependent. Thus it remains to be understood why some physical situations are more favourable than others to the recognition that velocities and distances travelled are different, and why velocities are more readily transformed in a frame change than distances. We sought answers to these questions through a number of experiments involving a total of some 700 students.

In each experiment, anonymous paper-and-pencil tests were given for some 20 minutes. Each test involved solving a qualitative exercise, with as few numbers and symbols as possible (to prevent the mechanical application of formulae and the intrusion of effects related to mathematical ability), dealing with uniform motion in two dimensions. (In more than one dimension, the directions of velocities and distances themselves differ from frame to frame, and this furnishes valuable information on how the problem is dealt with.)

Each experiment was centred on a definite exercise scheme, and the various tests in the experiment corresponded to various forms of the overall scheme. We could thus more easily reveal 'constants of motion' and relevant parameters.

By collating the information obtained, we were finally able to pinpoint a number of rules and relations describing SWR, the existence of which we had postulated. Although these rules and relations were not always evinced by the students, they do account coherently for all answers and thereby form an organised system of thought which we term the 'natural model†', as opposed to the kinematic model of the physicist. This model is in fact rarely found in its pure state: most often, students will stand somewhere between the natural and kinematic models. Moreover, some elements of the natural model are even found in the speech of professional physicists—convincing evidence of its existence.

4 Description of the 'natural model'

The 'natural model' is quite complex, because it involves two components which always interact: a purely descriptive one describing motion, and a causal one explaining motion. In other words, kinematic and dynamic considerations are not clearly separated and distinguished as they are in conventional mechanics.

4.1. The causal component of the natural model

This component stems from the permanence of a link between motion and its causes. Motion is thus endowed with an intrinsic existence and tends to be defined through reference to the driving forces which cause it. These forces are in turn considered to be permanent physical properties of the moving bodies, independent of observers: we shall emphasise these characteristics by calling such forces 'proper driving forces'.

Velocities are only very loosely defined with respect to frames. This probably corresponds to the fact that, in everyday life, a particular frame is very often highly favoured for practical reasons (the ground being the most common example). The need to refer to a frame thus disappears, together with the need for or possibility of changing frames. Thus the velocity, instead of appearing as a property of the pair (moving body + frame), naturally becomes a property of the moving body alone, an intrinsic characteristic amongst others; again this is often emphasised by the use of the epithet 'proper'.

This concept of motion and velocity has a

† This does not mean that this model will necessarily lead to wholly coherent answers, as would an actual theory. It only means that it permits a coherent explanation of the answers as a whole.

number of consequences:
(i) In many problems, proper velocities are exclusively involved, no effort being attempted to define a frame.

The following exercise is typical: Observer A is stationary on the bank of a lake; Observer B is in a boat moving at constant speed parallel to the bank (figure 1). Looking towards the boat, B throws a

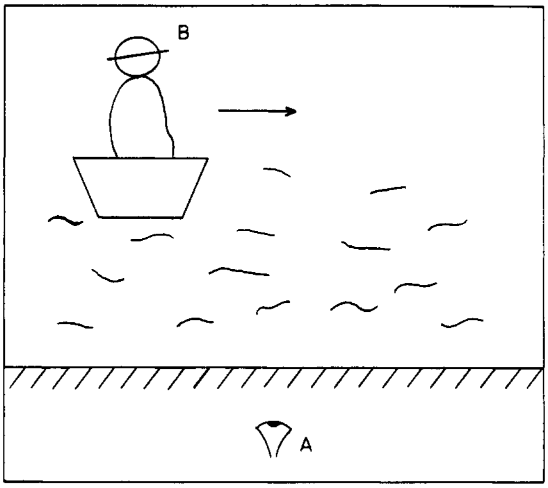

Figure 1

stone straight in front of him. A receives the stone and throws it back to B, who is hit. Compare the times of flight of the stone from B to A and back.

The data are of course insufficient to solve the problem. (It is interesting to see whether students perceive the insufficiency, and what hypothesis, implicit or explicit, they adopt to circumvent it.) Most students suppose that 'throwing velocities' are the same for A and B, a reasonable and perfectly admissible hypothesis. But these velocities are conceived as 'proper velocities independent of frame', and many students fail to recognise that they pertain to distinct reference frames for A and B.
(ii) However, situations with drag exemplify the

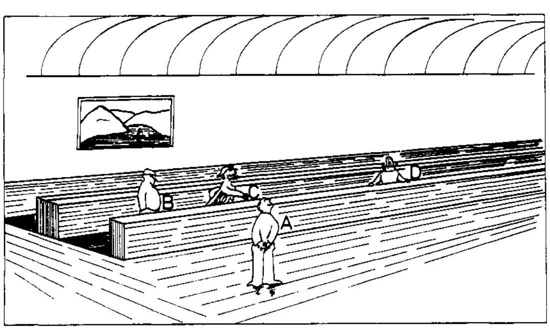

Figure 2

need to distinguish between different velocities for different frames.

A (stationary on the ground) and B (stationary on the moving pavement M) watch C walking on M (figure 2).

'For B, C has its proper velocity. For A, C has its proper velocity *plus* the velocity of M'.

Such answers are compatible with a kinematic composition of velocities. But this is not the reasoning path actually followed by most students, as is shown by the following test: The stream in river R has uniform speed everywhere. Two boats cross R at different orientations with respect to the banks (figure 3). Define the trajectories and calculate the crossing times.

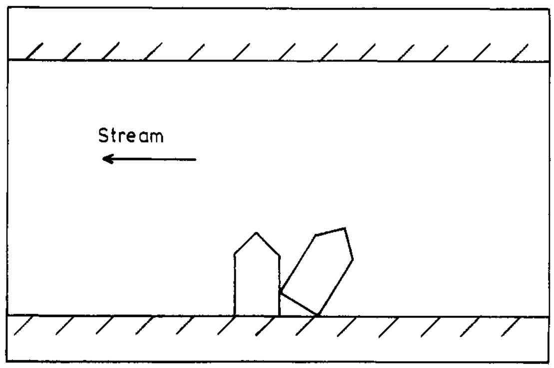

Figure 3

It appears here that the 'causes' of motion combine with one other as forces and not as kinematic velocities: the composition diagrams, for a boat starting perpendicular to the bank, frequently appear as in figure 4. Two kinds of trajectories appear (figure 5):
(*a*) Either a curve resulting from the progressive action of the stream—a correct (though probably not 'correctly' obtained) velocity is not sufficient to induce a correct trajectory.
(*b*) Or a straight line, correctly depicting the kinematic situation, but frequently with a justification which indicates a very limited mastery of the

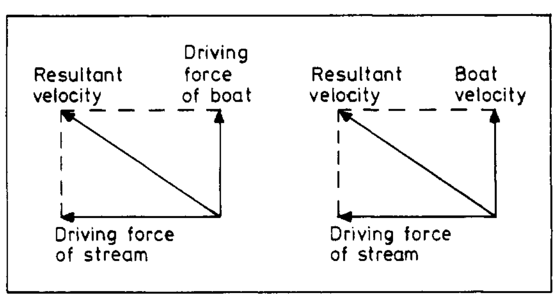

Figure 4

problem—for instance 'The trajectory results from the *simultaneous actions* of the boat and the stream'.

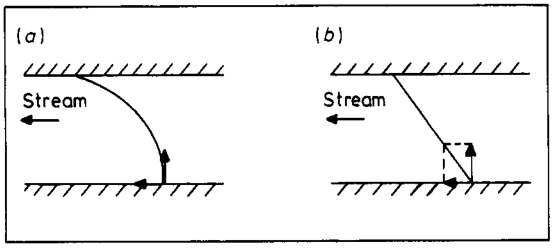

Figure 5

Thus figure 5(*b*) actually differs from figure 5(*a*) as a result of a different intuition regarding the influence of the stream on the motion of the boat: a progressive influence in the former case, an immediate one in the latter. But, in both cases, velocities are defined as proper velocities, determined by driving forces acting on the moving body. The stream here appears as a 'motor', a supplementary cause of the driving force acting on the moving body, and does not come into play through a kinematic composition of velocities, but essentially through a dynamical composition of forces or a composition of causes of motion.

(iii) Removing a drag will often result in the use of the proper velocity alone.

This statement, which strengthens our points (i) and (ii) above, can be made clear by the following test: Swimmer A is at rest, clinging to the bank of river R. Swimmer B drifts at constant speed in R. A and B observe a fish which is jumping, roughly parallel to the stream.

Students will often say: 'There is only one velocity for the fish. Once it is out the water, it no longer experiences the drag of the stream. Its velocity is thus the same for A and B'.

The stream again appears as a 'motor', and the presence of this supplementary 'motor' relieves the body of the necessity to move with a fixed 'proper' velocity determined by its 'proper motor'. On the other hand, when the link between the moving body and this 'dragging motor' disappears, the velocity is easily thought of as blocked at its proper value. In other words, when this link is broken, the drag velocity is not taken into account any further.

This component of the natural model proceeds from, and leads to, an inability to distinguish between purely kinematic aspects and dynamical ones. This inability also reveals itself in the field of dynamics (Viennot 1979): when the directions of the interaction forces do not coincide with the direction of the motion they produce, students frequently introduce an extra force proportional to the velocity. In both situations, velocities and forces are intimately mixed with one another, and, needless to say, we are led very far from galilean or newtonian mechanics.

4.2. The descriptive component of the natural model

This component refers to all properties pertaining to the relative positions of various moving bodies, irrespective of the origin of their motion. It involves two quite distinct aspects.

4.2.1. Geometrisation of dynamically defined motion, with the elimination of temporal considerations. This geometrisation essentially affects distances travelled and trajectories. In the stone-throwing test (figure 1) students are asked whether the distances of flight of the stone are the same for both observers: 60% answer that these distances are equal, because

'distances are independent of frames',

'the distance between two points is found to be the same whoever measures it',

'it is the same, apart from perspective effects which may come into play',

'time is the same, hence distance is the same'.

We may notice that reference is made to the positions and viewing angles of the observers. These observers are pinned to *fixed positions* when they are to make spatial measurements. Even if time is used to define their relative motion, it is no longer necessary when they proceed to such measurements. Once geometrical problems such as perspective or parallax effects have been corrected for, distances will emerge as identical for all observers. Some uncertainty may be apparent: 'B is sitting in a moving boat'; a correction then follows: 'The distance flown by the stone is the distance between A and B *when* B *is stopped*'. The elimination of motion and of time is felt to be necessary before distance measurements can be made.

It thus appears that the space where motion takes place must be *unique, independent of observers, and purely geometrical*; time then only appears to allow passing from one place to another.

This geometrisation has a number of consequences:

(i) The distance travelled is all the more 'intrinsic' when starting and arrival points are defined by physical objects in fixed relative positions. Students admit that C (figure 2) for one step travels a greater distance for A than for B because 'when C makes a step, the moving pavement M also pushes him forwards'. But if C goes from B to another man D, also stationary on M, they claim that the distance he travels is unique and the same for B and A. In the same way, they all consider that one can define only one distance travelled by C between the time he steps onto M and the time he steps off, this distance being the length of M. (It may be remarked, in this respect, that a first quick answer of a similar kind is often obtained from physicists.

When they correct it, it is through reference to some $d = vt$ relation. This kind of reasoning is not at all spontaneous.)

(ii) As positions of moving bodies are fixed prior to any discussion of distances travelled, the distinction between these and instantaneous distances (separating positions of moving bodies taken at the same instant) tends to be blurred. This is a source of new difficulties.

We have said that geometrisation also affects trajectories: these tend to become well-defined curves which can be transferred as a whole and without modification from one frame to another (Saltiel 1978, Viennot 1979).

Nine students out of ten (Malgrange and Maury 1976) think that a heavy body (a lead ball for instance) thrown upwards by a man on a moving pavement will fall behind him because

'when the ball is in the air, there is no physical link left between it and the pavement',

'the ball *loses* its horizontal speed *instantaneously*',

'the ball must go up and down along *the vertical*'.

The first two 'arguments' are of the dynamical type; the last one is geometrical: there is only one possible absolute trajectory for a falling body, the vertical, which becomes a sort of rigid object, like a rule, which can be transferred from frame to frame. Such geometrical and dynamical arguments are often used interchangeably, and reinforce one another within a quite coherent natural model.

4.2.2. Viewpoints of various observers. The preliminary investigation had shown that students admit rather easily the variations of velocity from frame to frame. However, we have seen that motion is intrinsically defined as a property of the moving body and not of observers. How can we explain this apparent contradiction? The key to this riddle is to be found in the 'well-known' distinction between 'real' (or 'true') and 'apparent' motions. To quote a student: 'Real motion takes place physically'. It has a recognised dynamical cause, for instance 'proper' motion, drag motion, and motion resulting from their composition. Only this type of motion appears in calculations and justifications, while 'apparent' motion is never used. This 'apparent' motion is merely an 'optical illusion, devoid of any physical reality' (again a student's quotation). It corresponds to a visual appearance which cannot be related to driving forces actually acting on the moving body. As such, it is all the more admissible that reference can be made to practical situations where such motion has been observed, but students are generally very far from showing a genuine kinematic understanding. For instance, the 'apparent' motion of the poster on the wall for people on the moving pavement of figure 2 is readily accepted: this agrees with daily experience. Now, if parachutist A drops his goggles G, while parachutist B has a greater vertical velocity than A, students will not easily recognise that the velocities of G for A and B are different. This recognition, however, will be immediately triggered off for those who are reminded of a film in which a falling object like G appears to a falling cameraman to travel upwards. But such phenomena are understood as delusions of perception, which allows them to coexist with 'real' motion and its intrinsic characteristics. Galilean relativity is completely ignored.

5 The natural model and its two kinds of 'invariants'

As shown by the reactions of first-year university students, the natural model displays two kinds of invariants: dynamic ones, that is causes of motion yielding 'proper velocities', and geometrical ones, that is distances travelled linked with positions (after time has been eliminated).

It was important to know whether these representations existed before physics teaching, and to see to what extent they survived for older students.

First, eleven-year old pupils were presented with situations where, owing to their own displacement in a rolling armchair, they perceived the 'apparent' motion of a pencil as being in the opposite direction to the 'real' motion of the pencil with respect to the room. Students strongly refused to accept such a situation. However, difficulties were much reduced when only the initial and final points of the pencil could be seen, and not its motion between them (Maury *et al.* 1977).

Second, a new test was proposed to first- and fourth-year university students, which deals with one-dimensional motion. An 'observer boat' O finds that two other boats a and b have, in his frame, positions A and B at some instant t, and positions A′ and B′ at a later instant t'. Between t and t', A has overtaken B. The situation is described in various frames, and the student must say whether these descriptions are compatible with the account given by O.

Two graphical representations of the observations of O are offered (lines 1 and 2 of figure 6). The first (named the 'motion test') more readily suggests the reconstruction of the motion of both boats in detail. The second (named the 'position test') tends to eliminate the motion of the boats between t and t', and to focus on their instantaneous relative positions and distances.

In the 'motion test', three supposed accounts of the motion of the boats in various frames are

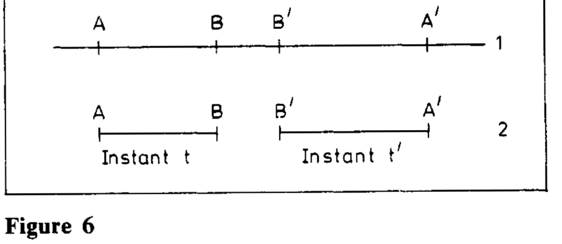

Figure 6

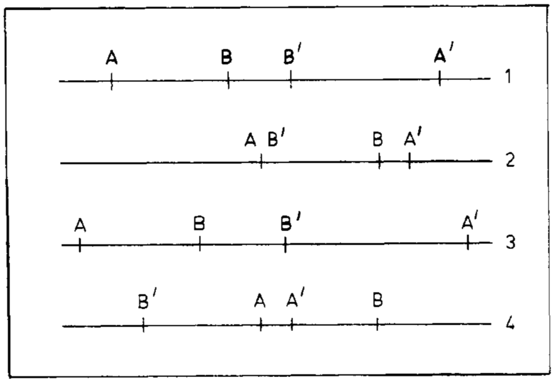

Figure 7

depicted (lines 2, 3 and 4 of figure 7), and their compatibility with the account of O (line 1 of figure 7) must be tested. This is done on the basis of the necessary invariance of AB and A′B′ under a change of frames, which shows that 2 and 4 are admissible, while 3 is not. The results obtained are presented in table 1. They are clearly poor, even

Table 1 Percentage of wrong answers in the motion test.

Account number	First-year students	Fourth-year students
2	75%	74%
3	75%	80%
4	75%	64%

for the older students. The criterion they often explicitly use refers to the directions of the motion: it is felt that a and b should always move in the same direction as they do for O (account number 1). We encounter again the 'direction invariant' (of dynamical origin) obtained with much younger pupils.

On the other hand, when the boat positions are given as in line 2 of figure 6, 90% of first-year students consider that these positions have intrinsic values and give the correct answer for accounts 2, 3 and 4: we observe the same phenomenon as with school pupils.

6 Summary of the natural model: pedagogical implications

The main characteristics of the natural model, compared with those of the 'correct' kinematic model, are summarised in table 2.

The fundamental differences between these models have several important consequences from the pedagogical viewpoint. We should like to stress the following:

(i) Almost never do students think in terms of frames, because they feel no need to do so. They feel free to combine kinematic quantities pertaining to different frames without any special care.

(ii) Forces and velocities are often intermixed, even when all motion is uniform. Our results, together with those of co-workers (Viennot 1979), point to the fact that velocities as well as forces exist '*per se*', independently of reference frames, and are endowed with causal properties, so that they are invoked to explain motion on a quite similar footing.

(iii) Proper motion and immobility are defined intrinsically, and not with respect to specified bodies or frames. Motion and rest are thus fundamentally inequivalent, a typical pre-galilean view. For example, the following sentences describing the same situation—'A has velocity v and B stands still' and 'A has velocity v with respect to B'—do not have an identical meaning for students. The first involves a causal aspect, and, contrary to the second, cannot be inverted into 'B has velocity $-v$ with respect to A'. Such an inversion, since driving or 'motor' forces are implied, would mean that A would be stopped and B physically set in motion. Moreover, this is all the more difficult since quantities are preferentially considered as positive, with the directions of velocities endowed with an intrinsic value much more strongly perceived than their absolute values: a car with velocity $-100\ \mathrm{km\,h^{-1}}$ has speed $100\ \mathrm{km\,h^{-1}}$ in the 'wrong' direction.

However, very often physics teaching both ignores the natural model and even helps to reinforce it by various imprudences.

In France, for instance, the composition law for velocities is still often expressed using the conventional vocabulary of 'absolute' or 'relative' velocities. This inevitably introduces a lack of symmetry between frames, and suggests that one velocity has a deeper, more physical meaning than the other (perhaps because it is more directly connected to a 'motor').

Exercises are often presented using a matter-of-fact vocabulary which bears no direct relation to the more abstract terminology used to express fundamental law, that is to define the kinematic model. A quotation from Zemansky and Sears (1970) will illustrate this point: 'A motor-boat is observed to travel at $16\ \mathrm{km\,h^{-1}}$ relative to the earth in the direction 37° north of east. If the velocity of the boat due to the wind is $3\ \mathrm{km\,h^{-1}}$ eastward and that due to the current is $6\ \mathrm{km\,h^{-1}}$ southward, what are the magnitude and direction of the velocity of the boat due to its own power?'. It is important that students become progressively able to use both languages and to translate from one into the other, but they should be trained and helped to bridge the gap.

Also, most exercises deal with situations involving drag effects, for which the natural and kinematic models present fewer divergences. It is important that other problems without drag be presented,

Table 2 Characteristics of the kinematic and natural models.

	Kinematic model	Natural model
Spatial aspects	Various 'spaces' corresponding to various frames with both spatial and temporal connections	*Unique geometrical space* without time reference, with various observers
Invariants	Durations, lengths, instantaneous distances, relative velocities	Durations, lengths, instantaneous distances and distances travelled, directions of displacements, 'proper' velocities
Composition laws	A unique law involving both temporal and spatial quantities, appearing under various aspects: composition of velocities, composition of distance travelled, variation of directions of motion with the frame	Two laws: a *causal* law (addition of driving forces and/or proper velocities) which involves resultant motion, and a *geometrical* law which involves 'apparent' motion

to accustom students to disconnect motion from driving forces. Situations with drag afford a transition between the natural and kinematic models. Situations without drag are necessary to come fully into the latter.

Graphical representations can easily be misleading: decomposition of a velocity vector (pertaining to a unique frame) into its components (vertical and horizontal, or radial and tangential) leads to a figure which may be readily confused with a velocity composition illustrating the change of velocity from one frame to another. It would be wise, in the latter situation, to adopt systematically a graphical representation with, as a first stage, distinct figures for distinct frames, and to stress explicitly the conventions involved when these figures are merged into a single figure.

Finally, classical mechanics is often presented with limited stress laid upon basic postulates. Only when relativity is reached are they discussed at any length, with frames and observers precisely defined, and due importance granted to the notion of an 'event', with its explicit temporal component. We suggest the introduction of the notion of instantaneous distance and its galilean invariance, and the composition law for distances travelled (for uniform motion), in order to counteract the 'natural' tendency to treat instantaneous distances and distances travelled on the same footing, and, more generally speaking, to eliminate the temporal aspect of motion in favour of its undue geometrisation.

It seems to us very important that students should perceive the contradictions between the natural and kinematic models: that they should recognise the incomplete and sometimes contradictory characteristics of SWR in order that a correct kinematic model takes its place.

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